HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 2011

EXTENSION 1 MATHEMATICS

TRIAL HIGHER SCHOOL CERTIFICATE

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General Instructions

- Reading time : 5 minutes
- **Working time : 2 hours**
- Attempt **all** questions.
- **Start a new answer booklet for each question making sure your student number is written at the top of each page.**
- All necessary working should be shown.
- This paper contains 6 questions worth 14 marks each. Total Marks: **84 marks.**
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and mathematical templates may be used.
- This examination paper must **not** be removed from the examination room.

Students Name:________________________

Teacher:________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2011 HSC Mathematics Examination.

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QUESTION 1. *Start a new answer booklet.*

a) Find the acute angle between the lines $2x - y = 0$ and $x + 3y = 0$ giving answer correct to the nearest minute. **2**

b) A is the point $(-2, -1)$, B is the point $(1, 5)$. **2**
Find the coordinates of the point Q, which divides AB externally in the ratio 5:2.

c) Solve $\dfrac{3x+2}{x-1} > 2$ **3**

d) $\cos A = \dfrac{3}{5}$, where $0 \le A \le \dfrac{\pi}{2}$ and $\sin B = \dfrac{5}{13}$, where $0 \le B \le \dfrac{\pi}{2}$

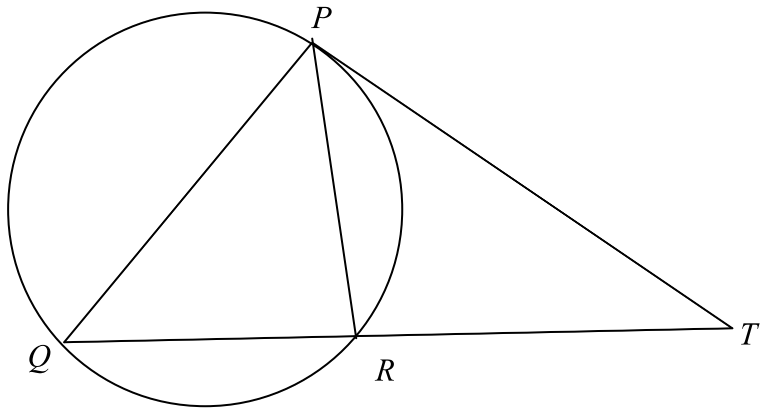
i) Show that $A = 2B$ **2**

ii) Find the exact value of $\tan(A + B)$. **2**

e) Show that $\dfrac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$ **3**

QUESTION 2. *Start a new answer booklet.*

a)



i) The Alternate segment theorem states **1**

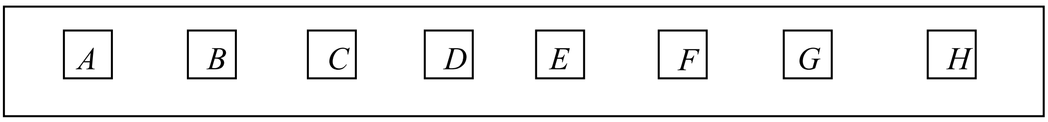
ii) PT is a tangent to the circle PRQ and QR is a secant intersecting the circle in Q and R. The line QR intersects PT at T.

Copy or trace the diagram

α) Prove that the triangles PRT and QPT are similar. **3**

β) Hence prove $PT^2 = QT \times RT$ **1**

b) A security lock has 8 buttons labeled as shown.



Each person using the lock is given a 3 letter code.

i) How many different codes are possible if letters can be repeated and their order is important? **1**

question 2 continued next page…

question 2 continued...

ii) How many different codes are possible if letters cannot be repeated and their order is important? **1**

iii) Now suppose that the lock operates by holding 3 buttons down together, so that order is not important. How many different codes are possible? **1**

c) Find the number of nine-letter arrangements that can be made from the letters in the word ***GLENFIELD***. **2**

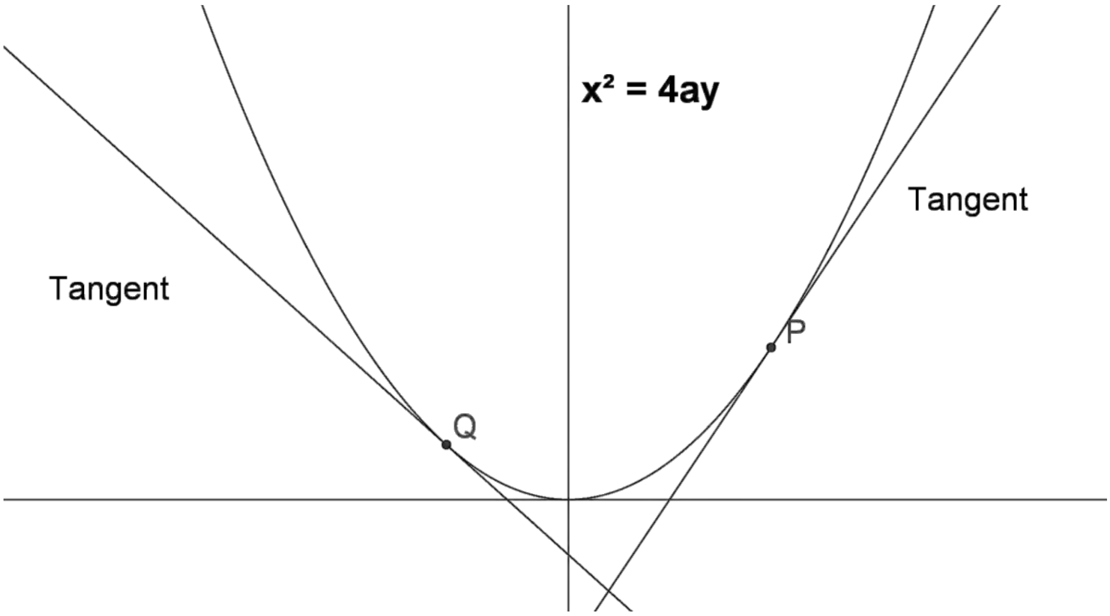
d) A meeting room contains a round table surrounded by ten chairs. These chairs are indistinguishable and equally spaced around the table.

i) A committee of ten people includes three teenagers. How many seating arrangements are there in which all three teenagers sit together? Give brief reasons for your answer. **2**

ii) Elections are held for the positions of Chairperson and Secretary in a different committee of ten people seated around this table. What is the probability that the two people selected are sitting directly opposite one another? Give brief reasons for your answer. **2**

QUESTION 3. *Start a new answer booklet.*

a)



$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.
The tangent at P and a line through Q parallel to the y-axis meet at point M.
The tangent at Q and the line through P parallel to the y-axis meet at N.

i) Prove that gradient of the tangent at P is p. **1**

ii) Prove that the equation of the tangent at P is $y = px - ap^2$ **1**

iii) Show that $PQMN$ is a parallelogram. **2**

iv) Show that the area of the parallelogram is $2a^2|p-q|^3$. **2**

v) Prove that the equation of the chord PQ is $y = (\frac{p+q}{2})x - apq$ given that its gradient is $\frac{p+q}{2}$ **1**

vi) Show that $pq = -1$ if PQ is a focal chord. **1**

vii) If PQ is a focal chord and $p = 2$, find the area of the parallelogram $PQMN$ in terms of a. **2**

question 3 continued next page…

question 3 continued…

b) $x^3 - 2x^2 - 11x + 12$ has roots α, β, γ. Without finding the values of these roots, find the values of:

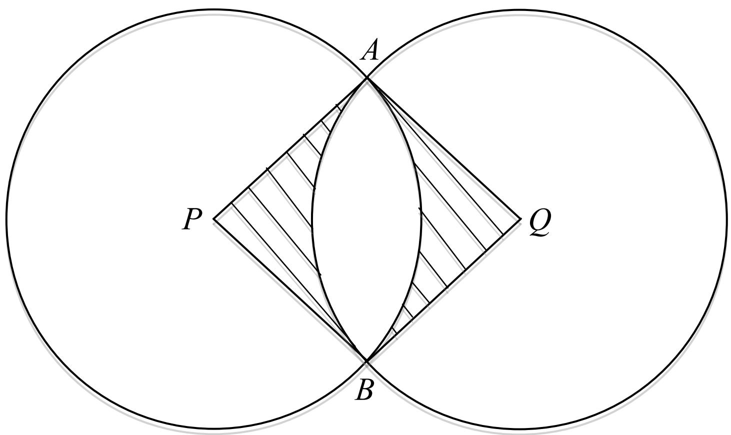
i) $\alpha + \beta + \gamma$ **1**

ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

QUESTION 4. *Start a new answer booklet.*

(a) Two circles centres *P* and *Q* respectively have radii 3 cm.
They intersect each other at *A* and *B*.
$AB = PQ$.



Find the area of the shaded region. **2**

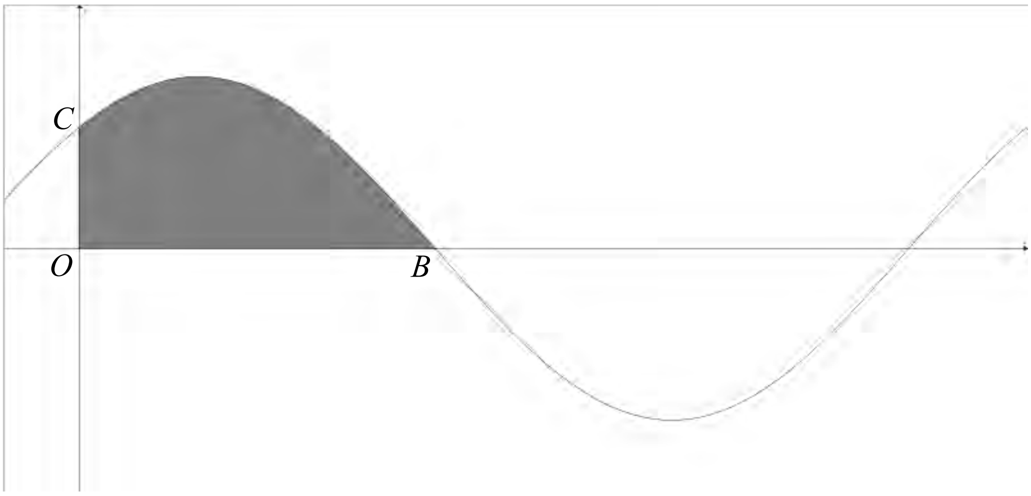
(b) Sketch the curve $y = 1 + 3\sin 2x$ for $0 \le x \le \pi$. **2**

(c) Find the equation of the tangent to the curve $y = \dfrac{\cos x}{x}$ at the point $\left(\dfrac{\pi}{2}, 0\right)$ **3**

question 4 continued next page…

question 4 continued…

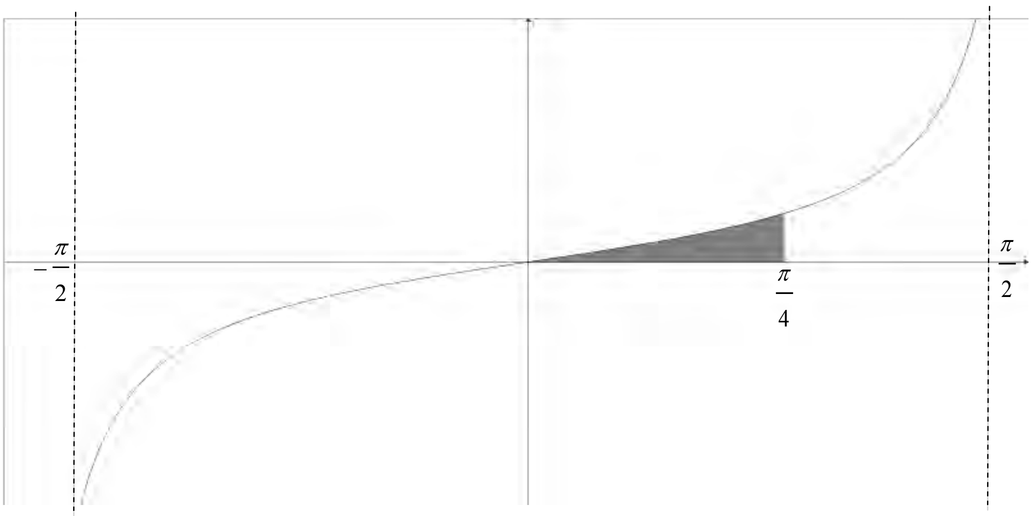
(d) The graph shows part of the curve $y = \sin x + \cos x$.



i) Find the coordinates of the points B and C. **2**

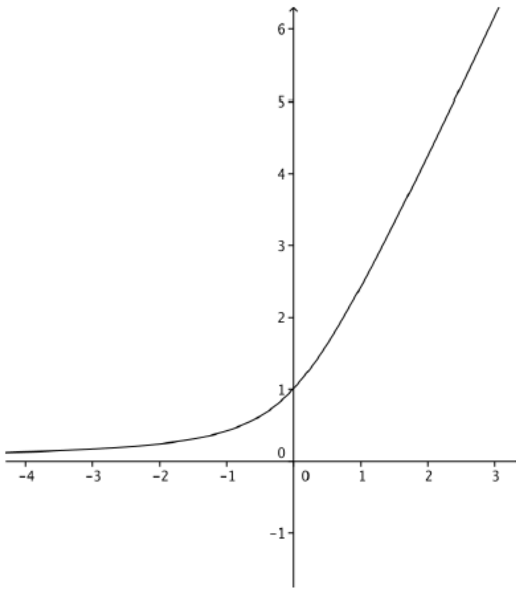
ii) Find the area of the shaded region. **2**

(e) The graph shows part of the curve $y = \tan x$.
The shaded region is rotated around the x – axis.
Find the volume of the solid of revolution. **3**



QUESTION 5. *Start a new answer booklet.*

(a) Consider the function $f(x) = x + \sqrt{x^2+1}$.



i) State the range and domain of $f(x)$. **1**

ii) Show that $f'(x) = \dfrac{f(x)}{\sqrt{x^2+1}}$ and, hence, show that $f'(x) > 0$ for all real x **2**

iii) State the domain and range of $f^{-1}(x)$, the inverse function of $f(x)$ **1**

iv) Show that $f^{-1}(x) = \dfrac{1}{2}(x - \dfrac{1}{x})$ **2**

(b) i) Find the domain and range of $y = 3\sin^{-1} 2x$ **2**

ii) Sketch the graph of $y = 3\sin^{-1} 2x$ **1**

(c) Evaluate $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{2-x^2}}$ **2**

(d) By writing $y = \tan^{-1}\sqrt{x}$ in the form $x = f(y)$, show that $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}\left(1+x\right)}$ **3**

QUESTION 6. *Start a new answer booklet.*

a) Evaluate $\int_0^1 2x(1-2x)^4 dx$ using the substitution $u=1-2x$. **3**

b) Using the substitution $t=\log_e x$, integrate $\log_e\left(x^{\frac{1}{x}}\right)$ with respect to x. **3**

c) Determine $\int \tan^2(x)\sec^2(x)\,dx$ by using the substitution $u=\tan(x)$. **2**

d) i) Factorise the polynomial $2n^2+7n+6$ **1**

ii) By use of the principle of mathematical induction, prove that: **3**

$$6\left(1^2+2^2+3^2+\ldots+n^2\right)=n(n+1)(2n+1) \quad \text{for} \quad n\geq 1.$$

iii) Hence, find the value of: **2**

$$\lim_{n\to\infty}\left(\frac{1^2+2^2+\ldots+n^2}{n^3}\right)$$

END OF EXAMINATION

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

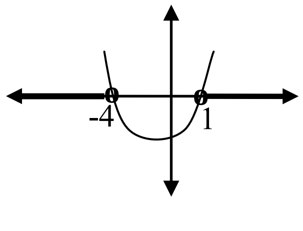
NOTE : $\ln x = \log_e x, \quad x > 0$

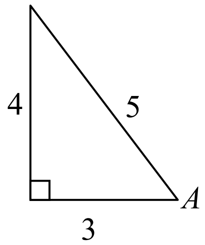
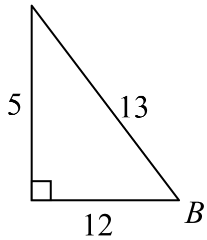
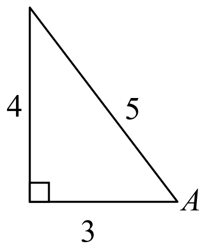
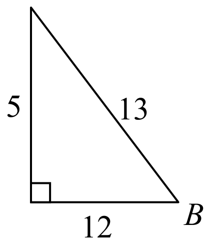
Year 12 Extension 1 Mathematics Yearly Examination 2011

Question No.1 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5-applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.
PE3-solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Solutions	Marking Guidelines
	1.	
H5	**a)** $2x - y = 0 \qquad x + 3y = 0$ $y = 2x \qquad y = -\frac{1}{3}x$	
	$m_1 = 2 \qquad m_2 = -\frac{1}{3}$	1 mark for finding correct gradients
	$\therefore \tan\theta = \left\lvert \frac{2 - \left(-\frac{1}{3}\right)}{1 + (2)\left(-\frac{1}{3}\right)} \right\rvert = 7$	
	$\therefore \theta = 81°52'$ (to nearest minute)	2 marks for complete correct solution
H5	**b)** $A(-2,-1) \quad B(1,5)$ External division ratio: $-5:2$	1 mark for partial correct solution
	$x = \frac{-2\times 2 + -5\times 1}{-5+2} = 3 \quad , \quad y = \frac{2\times -1 + -5\times 5}{-5+2} = 9$ $\therefore Q(3,9)$	2 marks for complete correct solution
PE3	**c)** $\frac{3x+2}{x-1} > 2 , \qquad x \neq 1$ $\frac{3x+2}{x-1} - 2 > 0$	
	$(x-1)(3x+2) - 2(x-1)^2 > 0 \quad$ [multiply through by $(x-1)^2$]	1 mark for one correct part of solution
	$(x-1)\left[3x+2-2(x-1)\right] > 0 \quad$ [factorising] $(x-1)(3x+2-2x+2) > 0$ $(x-1)(x+4) > 0$	2 marks for two correct parts of solution
		
	$\therefore$ Solution is $x < -4, \ x > 1.$	3 marks for complete correct solution

H5	**d)** **i)**   $\sin A = \frac{4}{5}$ $\quad$ $\sin B = \frac{5}{13}$ $\quad$ $\cos B = \frac{12}{13}$ Now, $\sin A = \frac{4}{5}$ $\quad$ $\sin 2B = 2\sin B\cos B$ $= 2\times\frac{5}{13}\times\frac{12}{13}$ $= \frac{120}{169}$ $\therefore \sin A \neq \sin 2B$ $\therefore A \neq 2B$	1 mark for partial correct solution. 2 marks for complete correct solution must indicate that $A \neq 2B$
H5	**ii)**   $\tan A = \frac{4}{3}$ $\quad$ $\tan B = \frac{5}{12}$ Now, $\tan\left(A+B\right) = \dfrac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \frac{63}{16}$	1 mark for partial correct solution. 2 marks for complete correct solution

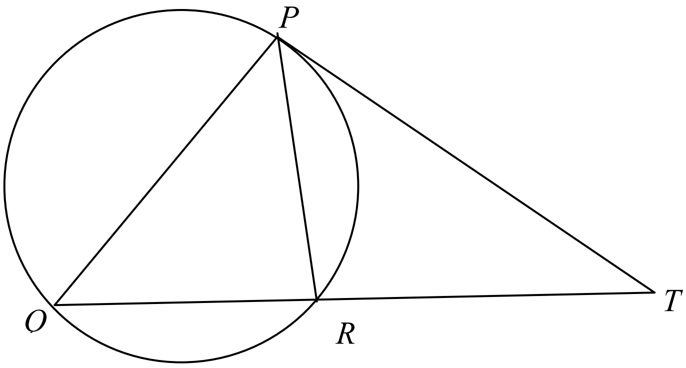
H5	**e)** $LHS = \dfrac{\sin 3x - \sin x}{\cos x - \cos 3x}$ $= \dfrac{\sin(2x+x) - \sin x}{\cos x - \cos(2x+x)}$ $= \dfrac{\sin 2x \cos x + \sin x \cos 2x - \sin x}{\cos x - (\cos 2x \cos x - \sin 2x \sin x)}$ $= \dfrac{\sin 2x \cos x + \sin x \cos 2x - \sin x}{\cos x - \cos 2x \cos x + \sin 2x \sin x}$ $= \dfrac{2\sin x \cos^2 x + \sin x(1 - 2\sin^2 x) - \sin x}{\cos x - (1 - 2\sin^2 x)\cos x + \sin 2x \sin x}$ $= \dfrac{2\sin x \cos^2 x + \sin x - 2\sin x \sin^2 x - \sin x}{\cos x - \cos x + 2\sin^2 x \cos x + \sin 2x \sin x}$ $= \dfrac{2\sin x \cos^2 x - 2\sin x \sin^2 x}{2\sin^2 x \cos x + \sin 2x \sin x}$ $= \dfrac{2\sin x(\cos^2 x - \sin^2 x)}{\sin x(2\sin x \cos x + \sin 2x)}$ $= \dfrac{2\sin x \cos 2x}{\sin x(\sin 2x + \sin 2x)}$ $= \dfrac{2\sin x \cos 2x}{\sin x(2\sin 2x)}$ $= \dfrac{2\sin x \cos 2x}{2\sin x \sin 2x}$ $= \dfrac{\cos 2x}{\sin 2x}$ $= \cot 2x$	1 mark for one correct part of solution 2 marks for two correct parts of solution 3 marks for complete correct solution

Year 12 Mathematics Extension 1 Half Yearly Examination Task 4 Trial HSC 2011

Question No. 2 Solutions and Marking Guidelines

Outcomes Addressed in this Question

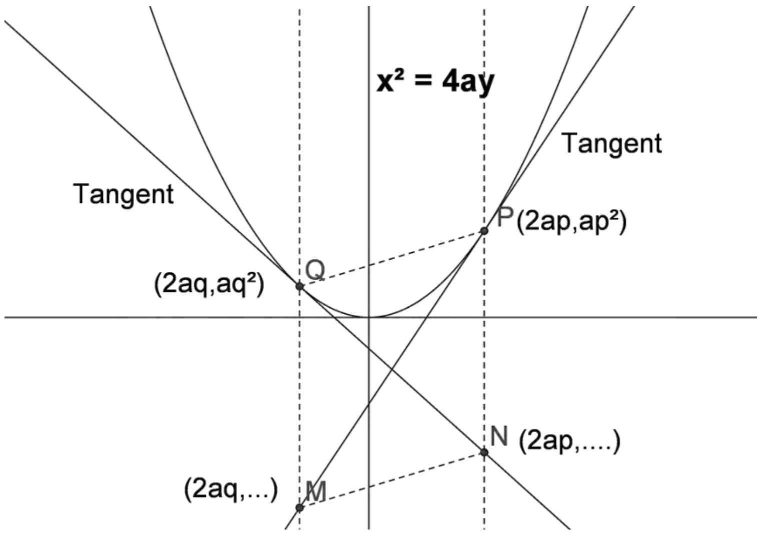
PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Solutions	Marking Guidelines
	(a) (i)The Alternate segment states that the angle between the chord and the tangent is equal to the angle in the alternate segment.	1 mark correct answer
		
	(ii)(α) In ΔPTR and ΔPQT $\angle TPR = \angle PQT\ (alternatesegmenttheorem)$ $\angle T$ is common $\therefore \Delta PTR\ /// \ \Delta PQT\ (equiangular)$	3 marks correct method leading to correct answer with reasons. 2 marks substantial progress towards correct answer with reasons. 1 mark some progress towards correct solution with reasons
	(β) $\frac{PT}{QT} = \frac{RT}{PT}$ (*corresponding sides similar triangles in proportion*) $\therefore PT^2 = RT \times QT$	1 mark correct reason
	(b) (i) $8 \times 8 \times 8 = 512$	1 mark correct answer
	(ii) $8 \times 7 \times 6 = 336$	1 mark correct answer
	(iii) ${}^8C_3 = 56$	1 mark correct answer
	(c) $\frac{9!}{2! \times 2!} = 90720$ different possible words	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
	(d) (i) Sit teenagers first 3! ways then sit remaining people 7! ways $3! \times 7! = 30240$ possibilities.	1mark correct answer. 1 mark correct reason.
	(ii) Sit chair person or secretary first, does not matter which, only 1 way this can happen. The other can sit in 8! Ways. Total possibilities 9! $P(E) = \frac{8!}{9!} = \frac{1}{9}$	1mark correct answer. 1 mark correct reason.

Year 12 **Mathematics Extension 1** **Assessment Task 4 2011**

Question No. 3 **Solutions and Marking Guidelines**

Outcomes Addressed in this Question: PE3, PE4, PE6,

Outcome	Solutions	Marking Guidelines
PE4	**(a)** 	***The diagram is not required.***
PE6	(i) The gradient of the tangent to the parabola is given by $\frac{dy}{dx}$ The parabola is $x^2 = 4ay$, or $y = \frac{x^2}{4a}$ $\frac{dy}{dx} = \frac{x}{2a}$ Substituting $x = 2ap$ into this equation, gives us $\frac{dy}{dx} = \frac{2ap}{2a}$ *i.e.* $\frac{dy}{dx} = p$	**1 mark:** complete
	(ii) The tangent has gradient p and passes through $(2ap, ap^2)$ Hence, $(y - ap^2) = (x - 2ap)p$ $y = ap^2 + px - 2ap^2$ $y = px - ap^2$	**1 mark:** complete
	(iii) PQMN is a parallelogram if PQ || MN and QM || PN QM || y-axis and PN || y-axis, $\therefore$ QM || PN Point M has coordinates $(2aq, 2apq - ap^2)$ Point N has coordinates $(2ap, 2apq - aq^2)$	**2 marks:** complete **1 mark:** one pair of opposite sides parallel.

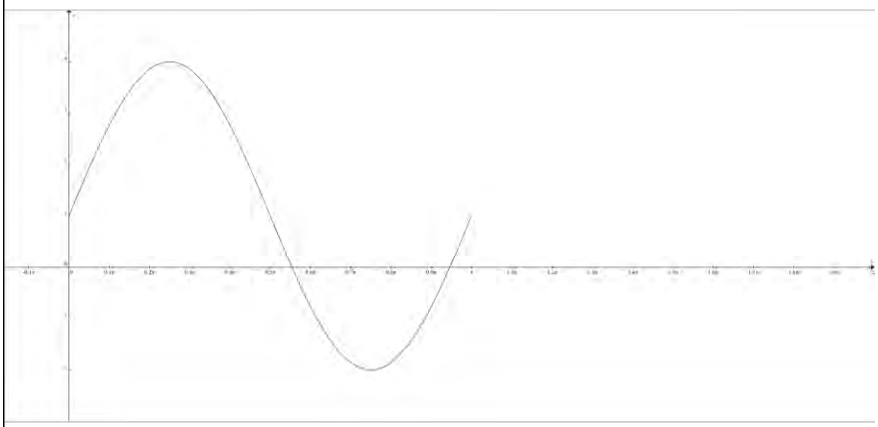
	Gradient of PQ = $\frac{p+q}{2}$ Gradient of NM = $\frac{2\text{apq-aq}^2-2apq+ap^2}{2ap-2aq}$ $=\frac{ap^2-aq^2}{2a(p-q)}$ $=\frac{p+q}{2}$ Hence, PQ || MN and PQMN is a parallelogram	
	(iv) Area of PQMN = $\lvert \text{PQ}\rvert \times \lvert QM \rvert$ $=\lvert[2a(p-q)]\times[ap^2+aq^2-2apq]\rvert$ $=2a^2\,\lvert(p-q)\times(p^2-2pq+q^2\rvert$ $= 2a^2\,\lvert p-q\rvert^3$	**<u>2 marks:</u> Complete solution** **<u>1 mark:</u> Some progress**
	(v) Chord PQ has gradient $\frac{\text{p+q}}{2}$ and passes through $(2ap, ap^2)$ The equation is $(y\text{ - }ap^2)=\frac{(p+q)}{2}\times(x-2ap)$ i.e. $y-ap^2=\frac{(p+q)}{2}x-ap^2-apq$ Hence, $\text{y} = \frac{(p+q)}{2}x-apq$	**<u>1 mark:</u>**
	(vi) If PQ is a focal chord it passes through (0,a) Hence a = 0 - apq, giving us pq = -1	**<u>1 mark:</u>**
	(vii) Area of PQMN = $2\text{a}^2\,\lvert p-q\rvert^3$ pq = -1, p = 2, q = $\frac{-1}{2}$ Area = $2\text{a}^2\left\lvert 2-\frac{-1}{2}\right\rvert^3$ Area = $2\text{a}^2(\frac{5}{2})^3$ *i.e.* $\frac{125a^2}{4}$ U^2	**<u>2 marks:</u> Complete solution** **<u>1 mark:</u> Some progress**

Outcome		Marking Guidelines
PE3	**(b)** $(i)\ \alpha+\beta+\gamma=2$ $(ii)\ \alpha\beta+\alpha\gamma+\beta\gamma=-11$ $(iii)\ \alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=4-2\times-11$ $=26$	**1 mark:** **1 mark:** **2 marks: Full solution** **1 mark: Some progress**

Year 12 Question No. 4	Extension 1 Mathematics Solutions and Marking Guidelines	Trial HSC 2011

Outcome Addressed in this Question

H5 **applies appropriate techniques from the study of calculus**, geometry, probability, **trigonometry** and series **to solve problems**

	Solutions	Marking Guidelines
(a)	$AQPB$ is a square $\therefore$ Area $AQPB = 9\,\text{cm}^2$ Area segment $PAB = \frac{1}{2}.3^2.\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)$ $= \frac{9}{2}\left(\frac{\pi}{2} - 1\right)$ = Area segment BAQ $\therefore$ Shaded Area $= 9 - 2.\frac{9}{2}\left(\frac{\pi}{2} - 1\right) = \left(18 - \frac{9\pi}{2}\right)\text{cm}^2$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(b)		**2 marks** ~ correct graph **1 mark** ~ correct graph but not enough detail
(c)	$y = \frac{\cos x}{x}$ $\frac{dy}{dx} = \frac{x.-\sin x - \cos x.1}{x^2} = \frac{-x\sin x - \cos x}{x^2}$ At $\left(\frac{\pi}{2}, 0\right)$, $\frac{dy}{dx} = \frac{-x\sin x - \cos x}{x^2} = -\frac{2}{\pi}$ Equation is $y - 0 = -\frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$.	**3 marks** ~ correct solution **2 marks** ~ substantial progress towards solution **1 mark** ~ limited progress towards solution
(d) (i)	$x = 0,\ y = 1 \ \therefore C(0,1)$ $y = 0, \sin x + \cos x = 0 \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}$ $\therefore B\left(\frac{3\pi}{4}, 0\right)$	**2 marks** ~ correct points **1 mark** ~ only one point correct

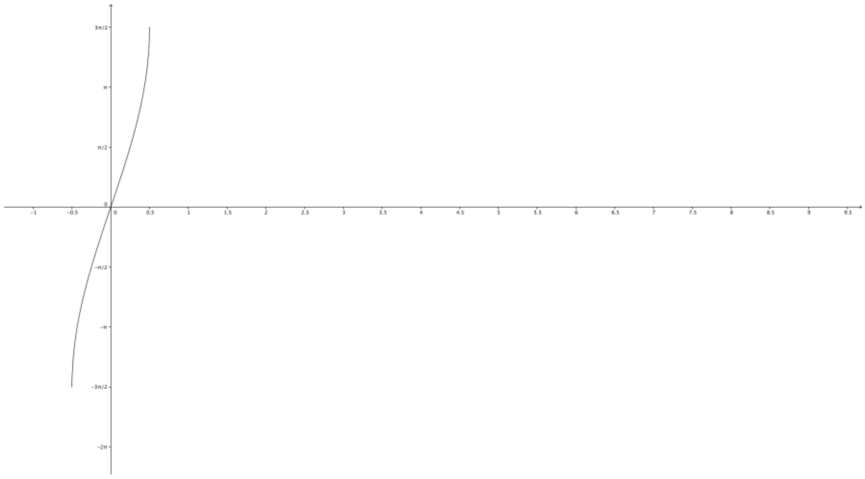
(ii)	$\text{Area} = \int_0^{\frac{3\pi}{4}} (\sin x + \cos x)\, dx$ $= \left[-\cos x + \sin x \right]_0^{\frac{3\pi}{4}}$ $= -\left(-\frac{1}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} - (-1 + 0)$ $= \sqrt{2} + 1 \quad u^2$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(e)	$V = \pi \int_0^{\frac{\pi}{4}} (\tan x)^2\, dx = \pi \int_0^{\frac{\pi}{4}} \tan^2 x \; dx$ $= \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 1)\, dx$ $= \pi \left[\tan x - x \right]_0^{\frac{\pi}{4}}$ $= \pi \left(1 - \frac{\pi}{4} \right) \quad u^3$	**3 marks** ~ correct solution **2 marks** ~ substantial progress towards solution **1 mark** ~ limited progress towards solution

Year 12 **Question No. 5**	**Extension 1 Mathematics** **Solutions and Marking Guidelines**	**Trial HSC 2011**

Outcome Addressed in this Question

HE4 uses the relationship between functions, inverse functions and their derivatives

	Solutions	**Marking Guidelines**
(a)(i)	Domain: $x \in °$ Range: $y > 0$	**1 mark** ~ correct domain <u>and</u> range
(ii)	$f(x) = x + \sqrt{x^2+1} = x + \left(x^2+1\right)^{\frac{1}{2}}$ $f'(x) = 1 + \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}.2x$ $= 1 + x\left(x^2+1\right)^{-\frac{1}{2}}$ $= 1 + \frac{x}{\sqrt{\left(x^2+1\right)}}$ $= \frac{\sqrt{x^2+1}+x}{\sqrt{\left(x^2+1\right)}}$ $= \frac{f(x)}{\sqrt{\left(x^2+1\right)}}$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(iii)	Domain: $x > 0$ Range: $y \in °$	**1 mark** ~ correct domain <u>and</u> range
(iv)	$f(x):\ y = x + \sqrt{x^2+1}$ $f^{-1}(x): x = y + \sqrt{y^2+1}$ $\sqrt{y^2+1} = x - y$ $y^2 + 1 = \left(x-y\right)^2 = x^2 - 2xy + y^2$ $\therefore x^2 - 2xy = 1$ $\therefore 2xy = x^2 - 1$ $\therefore y = \frac{x^2-1}{2x} = \frac{1}{2}\left(\frac{x^2-1}{x}\right) = \frac{1}{2}\left(x - \frac{1}{x}\right)$ $\therefore f^{-1}(x) = \frac{1}{2}\left(x - \frac{1}{x}\right)$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(b) (i)	Domain: $-\frac{1}{2} \le x \le \frac{1}{2}$ Range: $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$	**2 marks** ~ correct domain <u>and</u> range **1 mark** ~ correct domain <u>or</u> range

(ii)		**1 mark** ~ correct graph
(c)	$\int_{-1}^{1}\frac{dx}{\sqrt{2-x^2}}=\left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_{-1}^{1}=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ $=\frac{\pi}{4}+\frac{\pi}{4}$ $=\frac{\pi}{2}$	**2 marks** ~ correct solution **1 mark** ~ substantial progress towards solution
(d)	$y=\tan^{-1}\sqrt{x}$ $\sqrt{x}=\tan y$ $\therefore x=\tan^2 y$ $\therefore \frac{dx}{dy}=2.\tan y.\sec^2 y$ $=2.\tan y.\left(1+\tan^2 y\right)$ $=2\sqrt{x}\left(1+x\right)$ $\therefore \frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}=\frac{1}{2\sqrt{x}\left(1+x\right)}$	**3 marks** ~ correct solution **2 marks** ~ substantial progress towards solution **1 mark** ~ limited progress towards solution

Year 12	Mathematics Extension 1	Assessment Task 4 2011
Question No. 6	Solutions and Marking Guidelines	

Outcomes Addressed in this Question: HE2, HE6

Outcome	Solutions	Marking Guidelines
Outcome		Marking Guidelines
HE6	(a) Let $u = 1 - 2x$. $\therefore du = -2dx$ and $dx = \frac{-du}{2}$ As $u = 1 - 2x$, $2x = 1 - u$. If $x = 0$, $u = 1$. If $x = 1$, $u = -1$ $\int_0^1 2x(1-2x)^4 dx = \frac{-1}{2}\int_1^{-1}(1-u)u^4 du$ $= \frac{1}{2}\int_{-1}^{1} u^4 du - \frac{1}{2}\int_{-1}^{1} u^5 du$ $= \frac{1}{2}\{\frac{u^5}{5} - \frac{u^6}{6}\}_{-1}^{1}$ $= \frac{1}{2}\{(\frac{1}{5} - \frac{1}{6}) - (\frac{-1}{5} - \frac{1}{6})\}$ $= \frac{1}{5}$	**<u>3 marks:</u> Complete solution.** **<u>2 marks:</u> Substantial progress.** **<u>1 mark:</u> Some progress**
HE6	(b) Let t = $\log_e x$, $\therefore dt = \frac{dx}{x}$ or $dx = x\,dt$ $\int \log_e(x^{\frac{1}{x}})dx = \int \frac{1}{x}\log_e x\,dx$ $= \int t\,dt$ $= \frac{t^2}{2} + C$ $= \frac{(\log_e x)^2}{2} + C$	**<u>3 marks:</u> Complete solution.** **<u>2 marks:</u> Substantial progress including resubstitution** **<u>1 mark:</u> Some progress**
HE6	(c) Let u = tan x. $\therefore$ du = $\sec^2(x)\,dx$ $\int \tan^2(x)\sec^2(x)\,dx = \int u^2\,du$ $= \frac{u^3}{3} + C$ $= \frac{\tan^3(x)}{3} + C$	**<u>2 marks:</u> Complete solution.** **<u>1 mark:</u> Some progress**

HE2	(c) $(i)\, 2n^2+7n+6=(2n+3)(n+2)$ $(ii)\, P(n): 6(1^2+2^2+3^2+.....+n^2)=n(n+1)(2n+1)$ Test for n = 1 LHS $=6\times 1$ or 6 RHS = $1\times 2\times 3$ or 6 LHS$=$RHS, $\therefore$ true for n = 1 Assume true for n = k, i.e. $6(1^2+2^2+3^2+.....+k^2)=k(k+1)(2k+1)$ Test for n = k+1. i.e. does $6(1^2+2^2+3^2+.....+k^2+(k+1)^2)=(k+1)(k+2)(2k+3)$? LHS = $k(k+1)(2k+1)+6(k+1)^2$ $=(k+1)[2k^2+k+6k+6]$ $=(k+1)(2k^2+7k+6)$ $=(k+1)(k+2)(2k+3)$ $=$RHS P(n) is true for n = k+1 whenever it is true for n = k Summation: P(n) is true for n = 1 and is true for n = k+1 whenever it is true for n = k. Hence P(n) is true for n ≥ 1 $(d)\ Limit_{n\to\infty}(\frac{1^2+2^2+3^2+........+n^2}{n^3})$ $=Limit_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}\ \ (from\,(c))$ $=\frac{2}{6}\ or\ \frac{1}{3}$	**<u>1 mark:</u>** **<u>3 marks:</u> Complete solution.** **<u>2 marks:</u> Substantial progress** **<u>1 mark:</u> Some progress** **<u>2 marks:</u> Complete solution.** **<u>1 mark:</u> Some progress**